Rotating liquid mirror

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One fascinating topic in hydrodynamics involves liquid in a rotating vessel. Properties of water in a rotating vessel, in both the steady and the transient states, have been discussed by Goodman.[1] In this note we shall deal with one property of a rotating liquid surface: the ability of that surface to function as a concave mirror.

Sutton[2] produced a parallel beam of light by placing an automobile headlamp at the focal point of a parabolic mirror formed by rotating a pool of mercury. More recently, Grube[3] discussed the use of a rotating pool of mercury as a telescope mirror as early as 1909, and surveyed some techniques for making rigid parabolic mirrors by allowing various liquids to solidify as they rotate.

Use of a rotating fluid to form optical quality mirrors is an idea whose time has come. Maran,[4] in a recent survey article, described a new process for fabricating high-quality telescope mirrors of up to 315-in. diam by allowing a shallow pool of molten glass to cool as it rotates. In another very informative article, Fisher[5] has discussed some of the problems in present large telescope mirrors and their proposed solutions using the new techniques of spin-casting glass and spinning mercury.

The shape of a rotating liquid surface can be determined using elementary considerations. Referring to Fig. 1, the net force on an element of liquid of mass m at the surface, which is the vector sum of the gravitational force F_z and the centrifugal (reaction) force F_r, must be perpendicular to the liquid surface in the steady state. Accordingly, the tangent of the angle of the surface with respect to the horizontal at that point is

$$\tan\theta = \frac{dz}{dr} = \frac{F_r}{F_z} = \frac{m\omega^2 r}{mg} = \left(\frac{\omega^2}{g}\right) r, \tag{1}$$

where ω is the angular speed and g is the acceleration of gravity. Equation (1) can be integrated directly to obtain the equation of the surface,

$$z = \int_0^r \left(\frac{\omega^2}{g}\right) r\, dr \tag{2}$$

or

$$z = (\omega^2/2g) r^2, \tag{3}$$

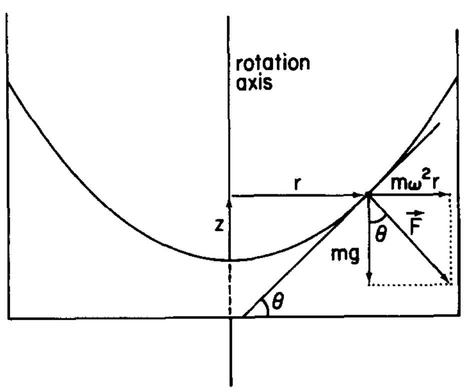

Fig. 1. Geometry for determining the shape of a rotating liquid surface.

where $z = 0$ is the level of the surface at the axis of rotation. This is the equation of a parabola,

$$z = r^2/4f, \tag{4}$$

where the focal length f of the parabola is

$$f = g/2\omega^2. \tag{5}$$

Because mercury is now considered a hazardous substance and can no longer be used in such a demonstration, we used glycerine. Water also works, but is not as viscous, so the glycerine is much more stable. Mixing dark dye into the glycerine reduces light reflecting other than at the parabolic surface, and therefore makes the observation cleaner.

One technique for investigating the focal properties of such a rotating liquid mirror is to provide a lighted object on the axis of rotation and observe the properties of the image formed. A convenient choice is the standard crossed arrow/pinhead object, which is asymmetric so inversion of the image can be observed.

We used a 45-rpm phonograph turntable to rotate the glycerine pool. The angular velocity is $4.7\ s^{-1}$ and the focal length is therefore 22 cm. Thus for an object approximately 70 cm above the rotating glycerine surface the image is real, inverted, and located 32 cm above the surface. Because this image is real, it can be observed by holding a ground glass screen at the image point. (Do not block all the light from the source.) Alternatively, Fig. 2 shows the TV camera

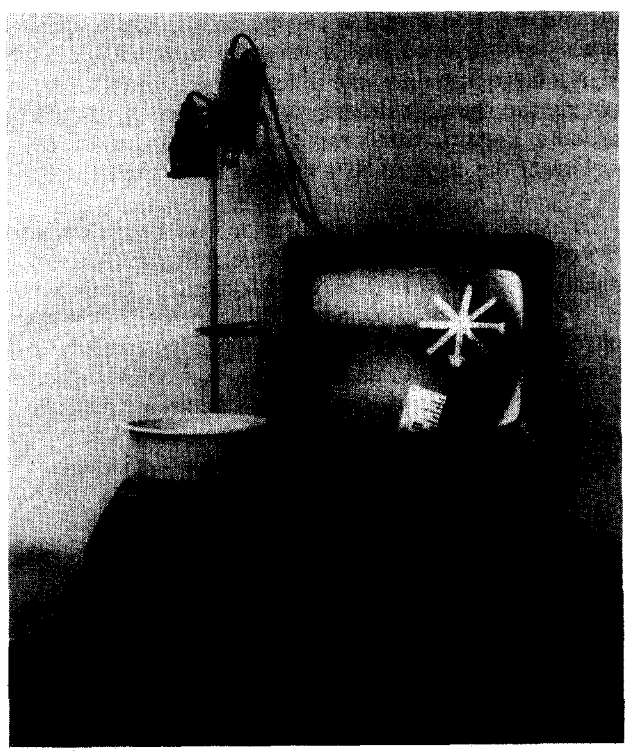

Fig. 2. Demonstration setup with the liquid rotating and the TV focused at the image plane of the concave mirror.

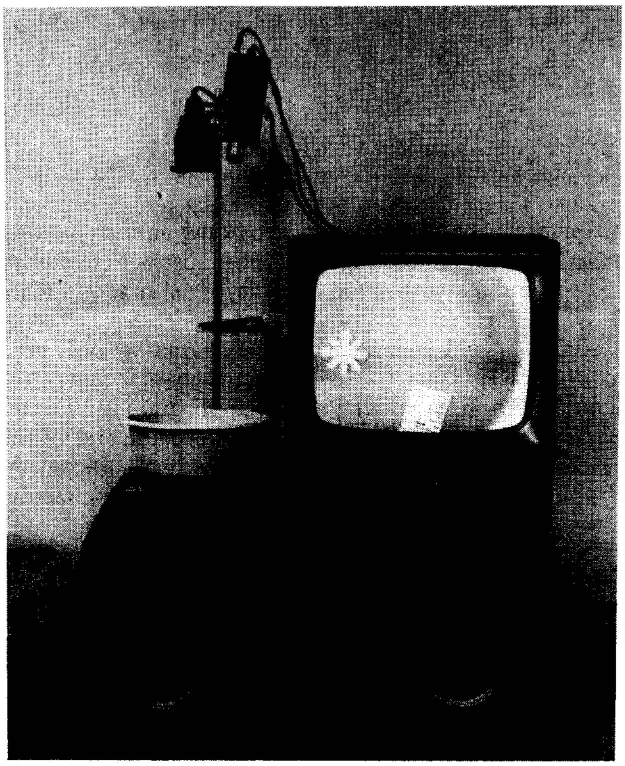

Fig. 3. Demonstration setup with the liquid quiescent and the TV focused at the image plane of the concave mirror.

focused onto the image plane for the rotating liquid mirror. A ruler at the image plane provides a reference with which the image can be compared.

When the turntable is at rest, the surface is a plane mirror with a virtual, erect image 70 cm below the surface of

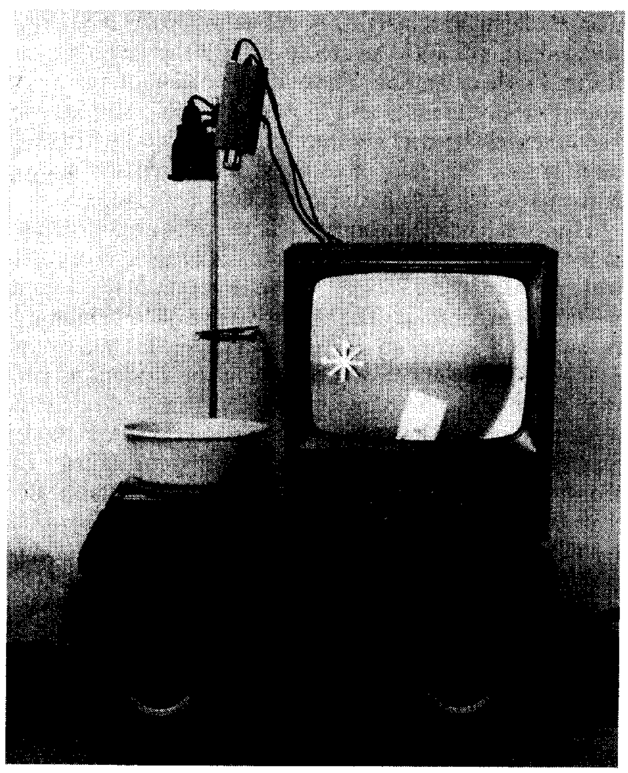

Fig. 4. Demonstration setup with the liquid quiescent and the TV focused at the image plane of the plane mirror.

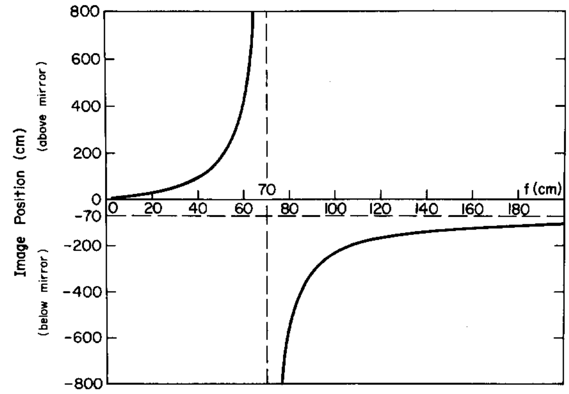

Fig. 5. Image position as a function of focal length of the rotating liquid mirror for a fixed 70-cm object distance.

the glycerine; Fig. 3 shows this situation with the camera still focused at the image plane for the curved mirror. In Fig. 4 the glycerine surface is also flat, but the TV camera has been focused onto the plane mirror image 70 cm below the glycerine surface, so the ruler is out of focus.

An interesting sidelight is the way in which the transformation occurs between the two types of image, which can be deduced with reference to Fig. 5. When the glycerine pool is started from rest into rotation, the focal length of the mirror surface decreases from infinity to 22 cm. The image moves from 70 cm behind the plane mirror away from the surface to infinity, when the focal length is 70 cm, the object distance. The image then reappears at infinity in front of the mirror (behind the TV camera) and moves slowly toward the mirror, to rest at the final image point 32 cm above the surface. Conversely, when the rotation ceases, the image moves out to infinity, then reappears at infinity behind the mirror, and moves slowly toward the surface until it is 70 cm behind the plane mirror. The image size is observed to increase as the image moves further from the mirror and to decrease as the image returns, in agreement with this theoretical picture. Also, the image is erect for the plane and very long focal length mirrors (lower right quadrant of Fig. 5), while it is inverted for the curved mirror whenever the object is outside the focal point of the mirror (upper left quadrant).

We have used this demonstration at the University of Maryland in classes and as a hands-on hallway experiment. In large classes it is necessary to use TV as described above, but for smaller classes and hallway use the image can be viewed directly. We leave the camera assembled as described in either case, because it is often helpful in the discussion and is interesting in its own right.

This demonstration is simple, but never ceases to elicit expressions of amazement, even from sophisticated observers.

[1] John M. Goodman, "Paraboloids and vortices in hydrodynamics," Am. J. Phys. **37**, 864–868 (1969).

[2] Richard Manliffe Sutton, *Demonstration Experiments in Physics* (McGraw-Hill, New York, 1938), Experiment M-143, Parabolic Mercury Mirror, p. 63, out of print but reprinted with permission by Kinko's Copies, 114 W. Franklin Street, Chapel Hill, NC 27516.

[3] Jack Grube, "Centripetal force and parabolic surfaces," Phys. Teach. **11**, 109–111 (1973).

[4] Stephen P. Maran, "A new generation of giant eyes gets ready to probe the Universe," Smithsonian Magazine **18**, 41–53 (1987).

[5] Arthur Fisher, "Spinning scopes," Pop. Sci. **231**(4), 76–79, 101–102 (1987).